# Kinematic Model of a Piano Action Mechanism

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## 1. INTRODUCTION

A modern piano action is a complex mechanism of wood, metal, and cloth (felt and leather) components that transfer forces applied by a pianist into a hammer motion that excites the string. In practice, an action must have an escapement system to allow the hammer to be free of the mechanism during the period of impact with the string, preventing blocking and the damping effect that this causes. In addition, a system to catch the hammer and allow rapid repetition is usually included. These two complexities may be disregarded when considering just the basic operation of the mechanism, which in essence is simply a system of three levers in series providing a rapid acceleration of the hammer from a small motion of the key.

Due to the difficulties arising from the non-linearities in the piano action dynamics, theoretical investigations initially focused on the kinematics [1]. More recent work has been able to incorporate some dynamic effects [2, 3]; in these models however, component parts are treated as rigid bodies, to simplify the mathematical representation, and action response is related only to the motion of these components in response to ideal, frictionless, mechanical linkages. Experimental investigations of action behaviour have been quite comprehensive [4, 5, 6, 7].

Despite the importance of dynamic effects, useful results can be obtained from a study of the kinematics alone [8]. This article presents a kinematic model of the (simplified) modern piano action and illustrates how non-trivial conclusions may be derived.

## 2. SYSTEM MODELLING

The basic action model consists of three components: the key, whippen/jack, and hammer, as shown in Figure 1. By ignoring the repetition mechanism, the whippen and jack have a fixed relationship and may be regarded as a single rigid body. When depressed, the key pivots about a point at A and, through the capstan connection, causes the whippen/jack to pivot about B. Through the knuckle connection, this causes the hammer to rotate about its pivot point C.

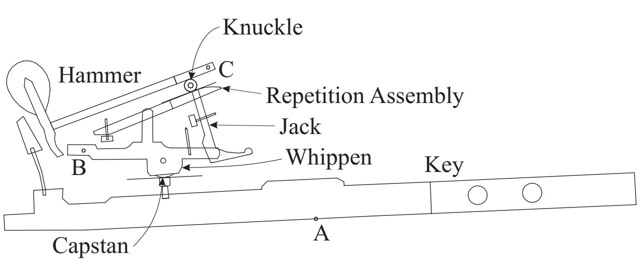


Figure 1. Schematic model of a piano action

All three pivot points, A, B, and C, are treated as ideal revolute joints. The connections at the capstan and the knuckle are sliding contact surfaces, modelled as prismatic and revolute joints in series. This simplified action can be seen in Figure 2.

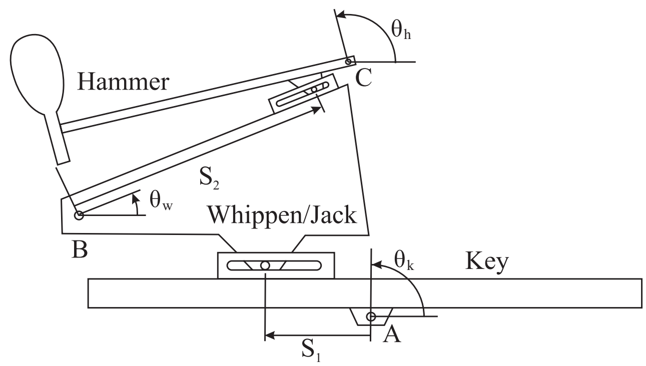


Figure 2. Simplified schematic of a piano action

A graph-theoretic approach [9] was used to automatically generate the closure conditions around the two independent kinematic loops. The result is a set of four nonlinear algebraic equations for the kinematics:

$$L_2 C_{\theta k} - L_3 C_{\theta w} - L_4 S_{\theta w} S_1 S_{\theta k} - L_1 = 0 \quad (1)$$

$$L_2 S_{\theta k} - L_3 S_{\theta w} + L_4 C_{\theta w} + S_1 C_{\theta k} - L_5 = 0 \quad (2)$$

$$L_8 C_{\theta h} - L_7 S_{\theta w} + L_9 S_{\theta h} + S_2 C_{\theta w} - L_6 = 0 \quad (3)$$

$$L_8 S_{\theta h} + L_7 C_{\theta w} - L_9 C_{\theta h} + S_2 S_{\theta w} + L_{10} = 0 \quad (4)$$

in which $L_1 - L_{10}$ are constants related to the dimensions of the components, $S_{\theta k}$, $C_{\theta k}$, $S_{\theta w}$, $C_{\theta w}$, $S_{\theta h}$, and $C_{\theta h}$ are the sines and cosines of the rotation angles of the key, whippen, and hammer, respectively, and $S_1$ and $S_2$ are the displacements of the two prismatic joints, as shown in Figure 2.

Thus, five variables appear in these four equations, as expected for a system with one degree of freedom. By prescribing a range of values for the key angle $\theta_k$, these nonlinear kinematic equations were solved using the Newton-Raphson method. Time was not considered in this analysis; instead, the relative positions of the components were determined for a given position of the key front (mm) below its rest position, which was easily calculated from $\theta_k$.

## 3. RESULTS AND DISCUSSION

One measurement that is of interest to piano designers is the extent of sliding between the contact surfaces at the capstan and knuckle. More sliding increases the wear of the components and reduces the efficiency of the action, due to the friction between the sliding surfaces. A plot of the relative movement between the two contact surfaces can be seen in Figure 3.

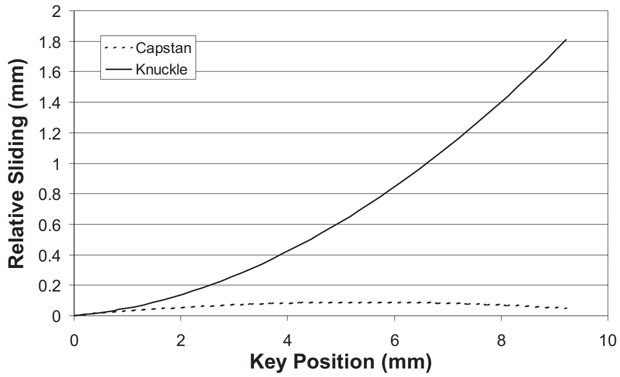


Figure 3. Relative sliding between components

There is very little sliding occurring at the capstan. While the point of contact between the key and whippen changes, the bodies are mostly rolling against each other, not sliding. Substantially more sliding occurs between the jack and hammer at the knuckle.

Another interesting measure that can easily be obtained from the model is the instantaneous effective leverage ratio of the action. Leverage ratio is defined as the ratio of the speed of the hammer to the speed of the key. This was approximated by calculating the ratio of the distance travelled by the hammer head and key front for each step of the solution.

In an effort to improve the response of a piano key, the geometry of the action might be changed by moving the capstan or the knuckle. Figure 4 shows the results of these changes, in which the kinematic equations were solved for three cases: (i) no change to capstan or knuckle; (ii) capstan shifted 10mm closer to the key front; and (iii) knuckle shifted 3mm closer to the pivot C.

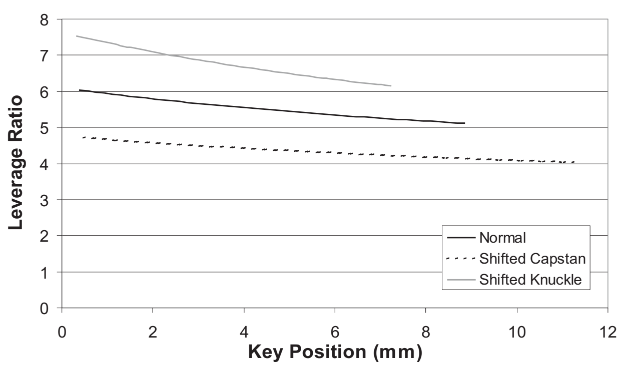


Figure 4. Leverage ratios

In all cases the leverage ratio decreases through the key stroke, meaning the key must be accelerated in order to maintain the same hammer speed.

Moving the capstan causes a significant decrease in the leverage ratio, resulting in a less responsive action and slower hammer speed. Shifting the knuckle increases the leverage ratio, which results in a higher hammer speed.

## 4. CONCLUSIONS AND FUTURE RESEARCH

Even with the simplified model used here, it is possible to obtain interesting results. With this information, a piano designer can determine where some of the frictional losses are occurring in the action, and quickly check how modifications to the design affect the relative motions of the components.

A more complex model could be obtained by representing some of the important dynamic effects. This would allow forces in the joints and between the contact surfaces, as well as component accelerations, to be calculated from a specified force input at the key.

In addition, the complexity of the component models could be increased by replacing the simplified ideal revolute joints with more realistic felt bushings, or using flexible beam models instead of some of the rigid bodies. The contact surfaces are currently modelled as ideal prismatic joints, but in reality they are contacts between rigid surfaces and cloth materials, for example wood and leather, or brass and felt. To account for the nonlinear, hysteretic nature of felt and leather, a more realistic contact dynamic model [10] could be incorporated.

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