Static and Dynamic Balancing of a Piano Key

Stephen Birkett[1]

Two Simple Cases

The basic principles of static and dynamic balancing can be illustrated[2] by representing the piano key as a massless beam resting on a central fulcrum. The action components - hammer, whippen, repetition assembly, and so on - are represented by a point mass m_1 acting on one end of the beam. An applied force F represents the finger acting at the other end of the beam to activate the key.

The key is balanced by adding a lead weight represented by a point mass m_2. Consider two simple cases: Case 1 - lead weight directly under the location of the finger force; and Case 2 - lead weight at a point between the finger and fulcrum.

Case 1 m_1, m_2, and F all act equidistant from the fulcrum (Fig. 1).

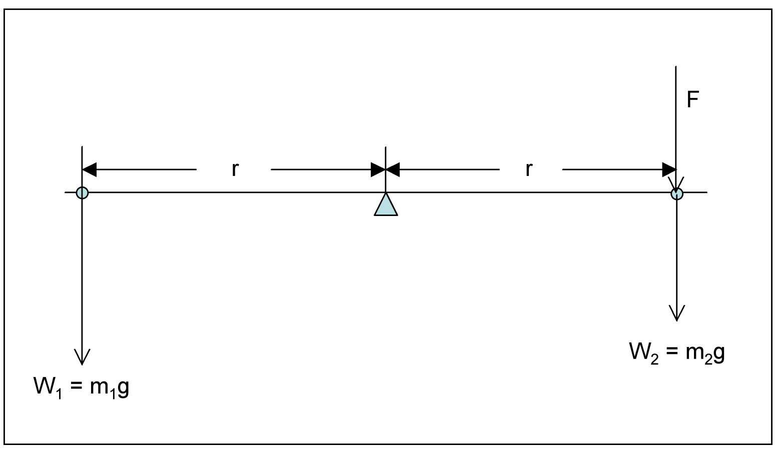

Figure 1: Activation force and lead weight act at the same point.

Fig. 2 shows acceleration of the key front (a) vs applied force[3] for the following scenarios: (i) the unbalanced key (red line), i.e. with no lead, for which the applied force has no effect until it exceeds the weight $W_1 = m_1 g$ of the hammer and other action components; (ii) a fully balanced key (dark blue line), for which any applied force causes an immediate acceleration; and (iii) a partially balanced key (light blue line), an intermediate static balancing.

The key with lead requires less force for the same degree of acceleration when $a < g$ (*soft zone*) and

[1]Department of Systems Design Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.
[2]Thanks to Martin Hirschkorn for the idea for this illustration.
[3]All of the graphs are based on the formulas given in the appendix obtained for the general case.

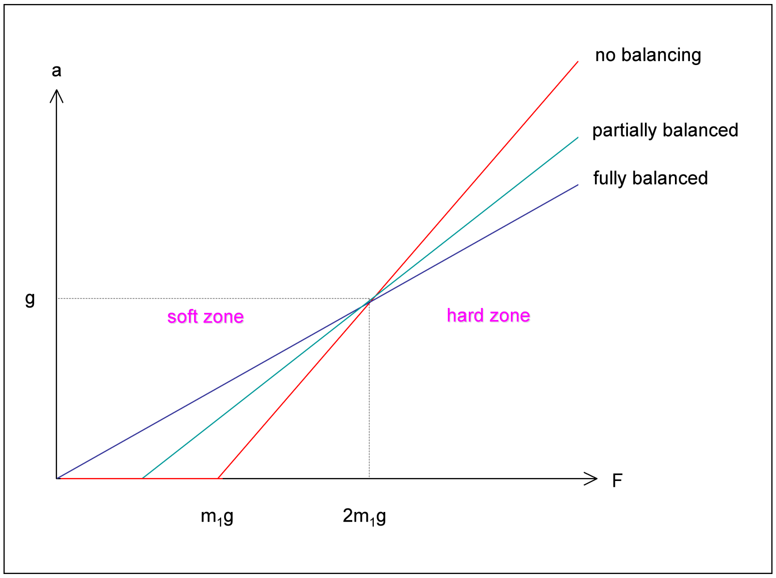

Figure 2: Acceleration vs force. Lead weight and activation force at the same point.

more force for the same degree of acceleration when $a > g$ (*hard zone*). A *dynamic breakpoint* occurs when the applied force is twice the weight of the action components W_1, at an acceleration $a = g$. Regardless of the mass of balancing lead used all of the graphs go through this same breakpoint when the location of the lead is the same.

In the soft zone the leaded key permits a greater range of force to be used to effect a smaller change in acceleration. In practice this may be considered to make the key less sensitive to variation in applied force, i.e. easier to control. Alternatively, the leaded key requires control of smaller absolute forces, which might imply that it is more sensitive, but harder to control. The interpretation of key control in the soft zone is therefore not unequivocal, and may depend on a pianist's degree of training, skill, and other factors. Experimental investigation would be interesting. In the hard zone the results are not ambiguous, since fine control becomes less of an issue and the leaded key is always physically more difficult to accelerate to achieve a given volume level, i.e. requires more force for the same volume.

Case 2 Activation force F and action component weight W_1 are equidistant from the fulcrum, and balancing lead weight W_2 acts at a variable point (Fig. 3). The case where the balancing lead weight is at midpoint between finger and fulcrum is shown in Fig. 4.

Compared to Case 1 the soft/hard breakpoint moves to a different location on the line corresponding to the lead-free unbalanced key (red line). The new breakpoint corresponds now to a force equal to $3W_1$ and acceleration 2g. Adding lead to a key closer to the fulcrum enlarges the soft zone. Moreover, there is less difference between the dynamic slopes for various static balanced conditions, i.e. the key behaves more like the unleaded key. All cases for which the lead is at the midpoint (e.g. the partially balanced key shown in light blue) will pass through the same breakpoint and have the same soft and hard zones. Moving the location of the lead moves the breakpoint (along the red line). Changing the mass of the lead at a fixed location changes the intercept (static balance force)

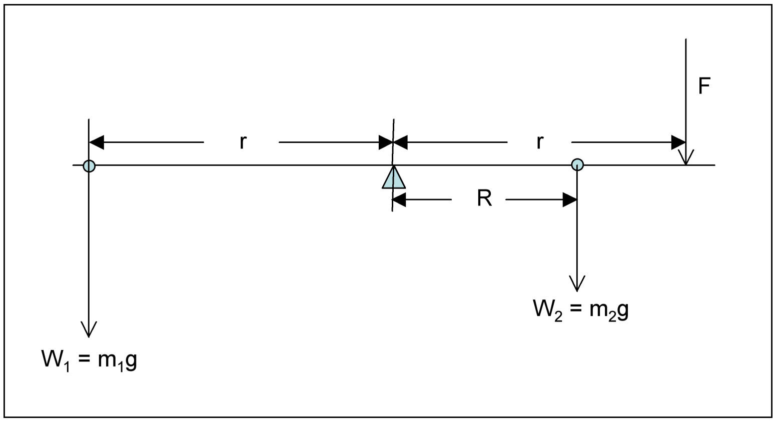

Figure 3: Activation force and lead weight act at different points.

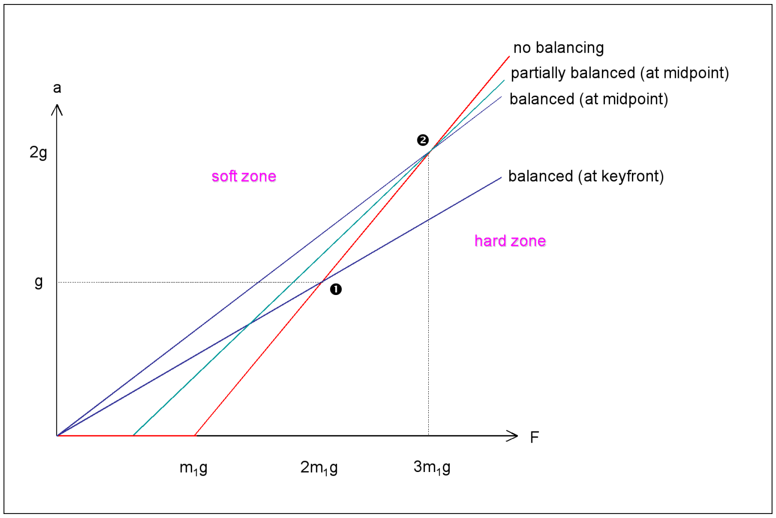

Figure 4: Acceleration vs force. Lead balancing weight at the midpoint between finger and fulcrum.

and consequently the slope of the line since the breakpoint does not change. This demonstrates that a key can be statically balanced with many different inertial or dynamic conditions.

An interesting theoretical case occurs under zero gravity conditions, for which static balancing is irrelevant and only inertial effects occur due to the masses. The breakpoint moves to the origin and there is no soft zone (Fig. 5).

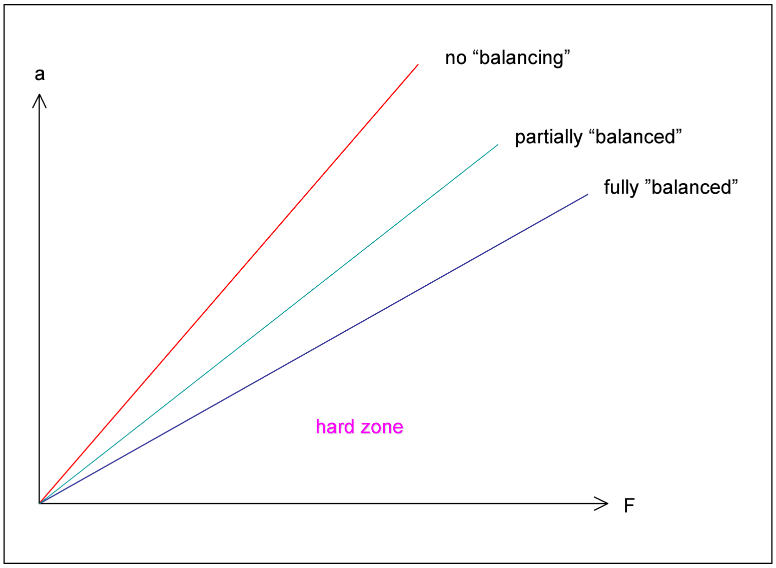

Figure 5: Acceleration vs force. Zero gravity conditions.

General case

Consider a real piano key and action components with distributed mass. This combination will be called the *(unleaded) reference key* and represented by a defined distributed mass beam with: (i) centre of mass (first moment about the fulcrum) and (ii) moment of inertia (second moment). A balancing lead mass m is added to the reference key at a defined location distance r_b in front of the fulcrum. The applied force acts at the front of the key, distance r from the fulcrum (see the diagram in Fig. 6).

Independent adjustment of both the first and second moments of the key+lead system is possible through two degrees of freedom, by varying both the mass and location of the inserted balancing lead. However, both of these factors must be changed simultaneously to achieve a given pair of pre-determined values for the two moments. Static balancing, and the concept of touch weight in conventional action regulation, considers only an adjustment to the first moment of the key and ignores the effects on the second moment. However the second moment is important for action dynamics and inertial effects.

Fig 7 shows general results for a given location of the balancing lead. This graph covers the following situations: (i) no balancing, i.e. the (unleaded) reference key with centre of mass behind

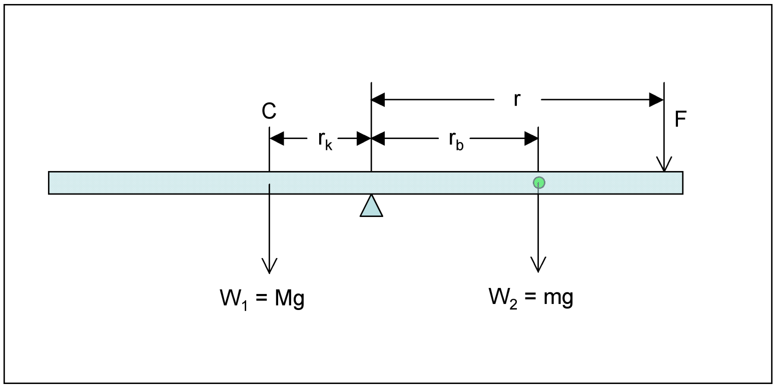

Figure 6: General case. Distributed mass key and balancing weight.

the fulcrum; (ii) partially balanced, i.e. lead has been added to move the centre of mass of the key+lead system closer, but still behind, the fulcrum; and (iii) fully balanced, i.e. sufficient lead has been added at the given location to move the centre of mass of the key+lead system to a point directly above the fulcrum location.

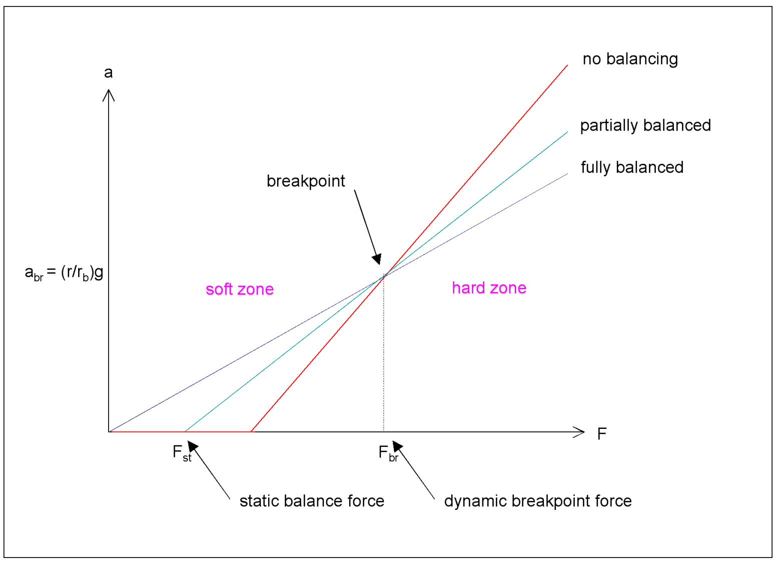

Figure 7: Acceleration vs force. General case of a distributed mass key and action components. Balancing lead at a defined location.

As in the simple cases, all the acceleration vs force graphs pass through the same breakpoint provided the lead is added at the same location on the key. The dynamic breakpoint lies on the acceleration vs force graph for the reference key at the point where acceleration is $a = (r/r_b)g$. This point defines the acceleration and corresponding applied force for which the reference and all leaded keys respond in exactly the same way. The existence of a breakpoint is a consequence of adding mass to the key at a point location (the key lead inserted into the body of the key stick). In general, adding lead closer to the fulcrum moves the breakpoint to higher acceleration and force values, giving a smaller range of possible acceleration vs force slopes, as well as slopes that are more similar to the slope for the reference key. In other words the dynamic effects from static balancing are minimized by inserting the lead as close as possible to the fulcrum.

Dynamic balancing

By changing the added mass(es) and their location(s) both the first moment and the second moment of the key can be independently adjusted. Inertial effects depend on the square of the distance to the fulcrum while static balancing depends on the distance, hence the divergence of their effects. The first moment determines the location of the static balance point (F_{st}), i.e. the intercept of the acceleration vs force graph; the second moment determines the slope of the graph, i.e. the dynamic relationship between applied force and acceleration of the key. Identical static balanced conditions can be achieved by varying either the mass or the location of the lead(s), but these can correspond to very different inertial situations. Simple static balancing as conventionally practised in action regulation[4] can provide similar (or graded) F_{st} values across a keyboard, but moments of inertia, and consequently dynamic sensitivity, are ignored and will essentially be random from key to key.

Simultaneous static and dynamic balancing requires careful simultaneous selection of both the lead mass(es) and their locations, so that similar (or graded) acceleration vs force slopes and intercepts can be achieved. The location of the lead determines the dynamic breakpoint when the inertial effects of the lead mass hinder (hard zone) acceleration instead of assisting (soft zone). This can be found as the intersection of the two acceleration vs force graphs, that for the reference key and that for the leaded key. Alternatively, if the location of the lead in the key is known, the breakpoint acceleration can be calculated as given by $a = (r/r_b)g$ and the breakpoint can be found on the graph corresponding to the reference key. With a known lead location the mass of lead used determines which line going through the breakpoint will be applicable to determine acceleration from applied force.

In practical terms, from the pianists control viewpoint the significance of different acceleration vs force gradients has yet to be examined experimentally, so at this time we cannot provide preferred values, or even confirm that dynamic balancing key to key is of practical importance. However, intuition and experience do suggest that dynamic variation is easily noticeable. The ‘assistance’ provided by a balancing lead in the soft zone can arguably be considered to make the key either more or less difficult to control in softer playing. The ‘hindrance’ from the lead in the hard playing zone is related to higher required forces for the same acceleration and volume level. As well as the dynamic effects in playing a key, after release the return of the key is also affected by inertia, with implications for repetition due to the added lead [heavily leaded keys feel sluggish].

Appendix. Derivation of dynamic equations for a leaded piano key

This appendix provides formulas that support the conclusions in the various graphs. The general case is analysed, from which the two simple cases may be obtained as well. For simplicity refer to the combined

[4] It is interesting to note that fortepianos NEVER had balancing leads in the keys, e.g. Viennese, English, and even early Erard double repetition actions.

(lead free) key stick and action components as the *reference key*. This corresponds to a piano action as it would occur in piano manufacturing prior to balancing. The following conventions and notation apply:

- Moment of inertia of the reference key is I_k.
- Centre of mass of the reference key is r_k, assumed to be behind the fulcrum point.
- Mass of the reference key is M.
- Mass of balancing lead is m, located distance r_b in front of the fulcrum point.
- Key activation force F acts at key front distance r in front of the fulcrum point.
- Acceleration of the key front is $a = r\alpha$, where α is the angular acceleration of the key.
- Torque on the key is τ.

The basic equation of motion is

$$\tau = I\alpha = \left[I_k + mr_b^2\right]\frac{a}{r}$$

Torque is derived from the activation force ($+Fr$), key weight acting at the centre of mass ($-Mgr_k$), and balancing lead weight acting at the balancing location ($+mgr_b$). This gives

$$Fr - Mgr_k + mgr_b = \left[I_k + mr_b^2\right]\frac{a}{r}$$

Solving for the force

$$F = \left[\frac{I_k + mr_b^2}{r^2}\right]a + Mg\frac{r_k}{r} - mg\frac{r_b}{r}$$

To locate the breakpoint, equate the two expressions for F applicable to two different balancing masses m. After cancelling terms and solving for a we get the breakpoint acceleration

$$a_{br} = \frac{r}{r_b}g$$

The corresponding breakpoint force is found by substituting this acceleration and simplifying:

$$\begin{aligned} F_{br} &= \left[\frac{I_k + mr_b^2}{r^2}\right]\frac{r}{r_b}g + Mg\frac{r_k}{r} - mg\frac{r_b}{r} \\ &= \left[Mr_k + \frac{I_k}{r_b}\right]\frac{g}{r} \end{aligned}$$

Note that the breakpoint force depends only on the properties of the key, as well as the locations of both the applied force and the lead weight. In particular, the breakpoint is independent of the mass of lead used to balance the key at a given fixed location.

In general, the static balance force (intercept $a = 0$) is

$$F_{st} = \left[Mr_k - mr_b\right]\frac{g}{r}$$

and the slope of the force vs acceleration graph is

$$\frac{\Delta a}{\Delta F} = \frac{r^2}{I_k + mr_b^2}$$

This slope is a measure of the dynamic sensitivity of the key to variation in applied force.